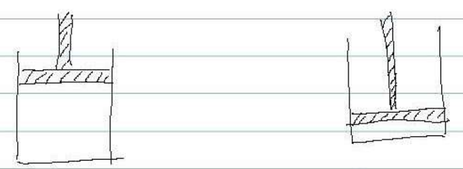
Chapter 18: Thermal Properties of Matter

Group Members:

1. Ideal Gas Law

   In an automobile engine, a mixture of air/gasoline is being compressed before ignition. Assuming the initial temperature of the air/gasoline mixture in the engine is $T_i = 25.0^{\circ}C$, find the final temperature of the air/gasoline mixture if the engine compression ratio is 1:9.

Compressing air in an automobile engine.



P_1, V_1, T_1 before

P_2, V_2, T_2 after

Given: $V_2 = \frac{1}{9}V_1$, $P_1 = 1 atm = 1.013 \times 10^5 Pa$

$P_2 = 21.7 atm = (1.013 \times 10^5 Pa)(21.7)$

$T_1 = 27^{\circ}C = (273 + 27) K = 300 K$

→ What is the final Temp T_2?

usage of the ideal gas law:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow T_2 = \frac{P_2 V_2}{P_1 V_1} T_1$$

T must be in K!

$$= \frac{(21.7 atm)}{(1 atm)} \frac{1}{9} (300 K)$$

$$= 723 K \text{ or } 450^{\circ}C$$

2. Molecular Properties of Matter

   a. How many moles are in a 1.00kg bottle of water? The molar mass of water is 18.0g/mol.

$$\# \text{ of moles} = (1.00 \times 10^{3}\, g) / (18.0\, g/mol)$$
$$= 55.6\ mol$$

   b. How many molecules?

   (The Avogadro's number is $N_A = 6.022 \times 10^{23}\, molecules/mol$)

$$\# \text{ of molecules} = (\# \text{ of moles}) \times N_A$$
$$= 3.35 \times 10^{25}$$

3. Kinetic Theory of an Ideal Gas

a. The surface of the sun has a temperature of about $5800K$ and consists of largely of hydrogen atoms. Find the rms speed v_{rms} of a hydrogen atom at the surface of the sun. (The mass of a single hydrogen atom is $1.67\times10^{-27}\,kg$ and the Boltzmann constant is $k=1.38\times10^{-23}\,J/K$.)

Treating the hydrogen atoms as a monoatomic Ideal Gas, there will be 3 degrees of freedom. Using the equipartition theorem, each dof will contribut $\frac{1}{2}kT$ to the average KE of the H atom in the gas,

So, $\langle KE\rangle = \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23}\,J/K)(5800K)$

$= 1.2\times10^{-19}\,J$

$\frac{1}{2}mV_{rms}^2 = \langle KE\rangle \Rightarrow V_{rms} = \sqrt{\frac{2\langle KE\rangle}{m}} = 1.20\times10^{4}\,m/s$

b. Assuming that the atmosphere of the Earth composes mainly of nitrogen molecules (N_2) and we are near the surface of the Earth where the temperature is about $25^\circ C(298K)$, what is the rms speed of a nitrogen molecule. (The molecular mass of a nitrogen molecule is $2.80\times10^{-26}\,kg$.)

The N_2 molecules will have 5 dofs but only 3 dofs are associated with its translational KE. The other 2 dofs are mainly related to its rotational motion.

So, for this gas, equipartition theorem will imply

$\langle KE\rangle$ per molecule $= \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23}\,J/K)(298K)$

$= 6.169\times10^{-21}\,J$

$V_{rms} = \sqrt{\frac{2\langle KE\rangle}{m}} = 664\,m/s$

c. Treating the nitrogen gas (N_2) as an ideal diatomic gas, what is the molar heat capacity C_V of the nitrogen gas?

Since N_2 has 5 primary dofs, so according to the equilibrium partition therem, each dof will contribute $\frac{1}{2}R$ to its C_V, so

$$C_v(N_2) = \frac{5}{2}R = \frac{5}{2}(8.314\ J/mol.K)$$
$$= 20.785\ J/mol.K$$

d. Now you replace the nitrogen gas (N_2) with an argon gas (*Ar*). Treating it as an ideal monatomic gas, what is then its molar heat capacity C_V ?

Ar gas is monatomic and it has 3 primary dofs,

so,

$$C_v = \frac{3}{2}R = \frac{3}{2}(8.314\ J/mol.K)$$
$$= 12.471\ J/mol.K$$